A Comparative Analysis of the Barbados and United States Business Cycles

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Abstract

This paper uses parametric and nonparametric methods to compare and contrast the characteristics of the different phases of the Barbados and U.S. business cycles. The two methods gave comparable results and indicated that the business cycles of these countries are closely linked, with Barbados cycle lagging behind the U.S. by six months or so. This result is attributed to the close trade relationship that exists between these two countries.

Keywords: Business cycles; Bry-Boschan Procedure; Markov Chains
I. Introduction

A considerable number of public, private and academic economists from around the world are once more devoting their attention to the study of economic cycles. This increased interest has been occasioned by several factors. One, it can be linked to the persistence, in many countries, of social imbalances such as unemployment and budgetary deficits, as well as to the growing consideration given to the important role of conjectural economic fluctuations and their consequences, when developing monetary and fiscal policies. Two, with the simultaneous deceleration observed in the United States (U.S.) and Europe between 2000 and 2001, and the current international context of globalized commercial trade and inter-penetration on national markets, the study of synchronized economic growth and of the mechanisms which propagate cycles has become evident (see, for example, Kose, Prasad and Terrones (2003), Heitz and Hild (2004)). Three, given the inadequacies of the early methods of decomposing temporal series into cyclical and trend-showing components, there was a need to develop an effective approach of correctly identifying, classifying and modeling economic cycles (Harding and Pagan (2002 a,b)).

Economic reviews like Backus and Kehoe (1992) show a significant diversity of studies carried out on developed countries. In addition, these reviews reflect a dearth of research on the theoretical modeling and on empirical knowledge of cycles in developing countries. This is disturbing, particularly since, a priori reasoning would suggest, that economic fluctuations are likely to be more substantial and persistent in developing countries than in industrialized economies.

The work presented in this article attempts to improve existing documentation on the phenomena of fluctuations and cycles in developing countries. Although some studies have been done along these lines (see Agenor, McDermott and Prasad (2000), Rand and Tarp (2002) and Du Plessis and Smit (2004)), their numbers remain relatively limited. In fact, for the small insular Caribbean states, empirical business cycle analysis is a very recent phenomena (see Craigwell and Maurin (2002, 2004,2005) and Cashin (2004)), and the focus has been on identifying, classifying and modeling economic cycles for individual Caribbean countries using various methods and data sets. In this paper, not only will the Barbadian business cycle be defined and
described but, for the first time, a comparison with the economic cycle of the United States (U.S.) will be presented utilizing the nonparametric and parametric methods of Bry and Boschan (1971) and Hamilton (1994), respectively.

The plan of this paper is as follows: Firstly, the link between the Barbadian and U.S. cycles is established and concepts and tools needed to compare national cycles discussed (Sections II and III respectively). Proceeding to the empirical applications, the nonparametric and parametric dating procedures are explained and the econometric estimates from these methods given (Section IV and V). Next, a comparison between the two countries cycles is made (Section VI). Finally conclusions are presented.

II. The Link Between Barbados and the U.S. Economy

Like other Caribbean countries, Barbados is a small island state with tiny internal markets, as well as limited endowments of natural resources and other production factors. As a result, the economy is very export-oriented and extremely dependent on its ties with industrialized countries like the U.S. Its growth experience over the last three decades has therefore been very much linked, on the one hand, to the potential to export implied by preferential agreements for access to large markets and, on the other hand, to increases in public expenditure funded through its institutional relationships with Europe and North America. However, due to this high degree of openness, the Barbadian economy is particularly vulnerable to shocks, especially exogenous shocks such as natural catastrophes, changes in the rules of engagement for accessing European and North American markets, fluctuations in global demand for its exports and varying levels of access to external financing.

Given the above, it is not surprising that fluctuations in Barbados real GDP follow those of the U.S. closely, although with a more pronounced magnitude and a delay of approximately one year (see Figure 1). Also tests for equality of means, medians and variances, found in EVIEWS 5, show that the two series do not differ significantly from each other. Moreover, instantaneous correlations \( (r) \) between the two countries, measured by \( t^* = r(l - r^2 / T - 2)^{-1/2} \), which follows
the student’s t-distribution with T-2 degrees of freedom, and calculated over different time periods, imply a significant correlation can be observed (see Table 1). All of these facts confirm the earlier result of Moore (2001), who utilising the Central Bank of Barbados’ forecasting model, indicated that if economic activity in the U.S. was to decline by 1%, real GDP in Barbados could decrease by approximately 0.5%, primarily because of the two countries link in tourism activity.

**Figure 1: Growth Rates of the Barbadian and U.S. GDPs**

![Graph of Growth Rates](image)

**Table 1: Correlation of Barbados and U.S. GDP growth rates**

<table>
<thead>
<tr>
<th>Period</th>
<th>Coefficient</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976-1989</td>
<td>0.49</td>
<td>1.95</td>
</tr>
<tr>
<td>1990-2003</td>
<td>0.64</td>
<td>2.88</td>
</tr>
<tr>
<td>1976-2003</td>
<td>0.54</td>
<td>3.30</td>
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</tbody>
</table>

The channels of commercial trade can also explain this close association between the Barbados’ cycle and that of the U.S. Indeed, publications on economic integration have shown that the more trade develops among countries, the greater the relationship between the different cycles (see, for example, Shin and Wang (2003)). Figure 2 as well as Tables 2 and 3, which depict the
The evolution of Barbados trade relations between 1960 to 2002, largely confirm this theory. Firstly, Barbadian exports to the U.S. increased almost 30-fold between 1970 and 1984, from $14.9 million to $416.5 million. There was a sharp decrease between 1985 and 1989, and since 1990, the figures have remained more or less stable at $70 million. Secondly, imports have also shown spectacular progress, expanding 13-fold between 1970 and 1984 from $49.3 million to $635.9 million. Between 1985 and 1992, there were a series of ups and downs, with an average value of $439.8 million. Imports doubled between 1993 and 2003. Finally, Table 2 shows that commercial trade in Barbados has undergone substantial changes in terms of their trading partners: Europe (more specifically, the United Kingdom), which was Barbados’ primary trading partner until the mid 1970s, lost this role to the United States during the 1980s. On average, almost 40% of Barbadian imports come from the United States.

In sum, the preceding facts reflect Barbados’ growing openness to and dependence on the U.S. Its growth and economic cycles are very much linked to supply and demand conditions as well as volumes and prices on the U.S. market. This means that there are an innumerable number of avenues, of varying degrees, for potential commercial crises, emanating in the U.S. market that can affect the Barbados’ economy.

**Figure 2: Direction of Trade between Barbados and the U.S. (BDS$ Million)**
Table 2: Direction of Barbados’ Trade Imports (BDS$ Million)

<table>
<thead>
<tr>
<th></th>
<th>Europe</th>
<th></th>
<th>U.S.</th>
<th></th>
<th>Canada</th>
<th></th>
<th>Caricom</th>
<th></th>
<th>Other Areas</th>
<th></th>
<th>Total Imports</th>
<th></th>
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</thead>
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<tr>
<td>Value</td>
<td>%</td>
<td>Value</td>
<td>%</td>
<td>Value</td>
<td>%</td>
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<td>Value</td>
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<tr>
<td>1960</td>
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<td>10.9</td>
<td>13.1</td>
<td>8.6</td>
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<td>13.2</td>
<td>12.1</td>
<td>14.5</td>
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<tr>
<td>1965</td>
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<td>11.9</td>
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</table>

Source: Barbados Statistical Service

Table 3: Direction of Barbados’ Trade Exports (BDS$ Million)

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<tr>
<th></th>
<th>Europe</th>
<th></th>
<th>U.S.</th>
<th></th>
<th>Canada</th>
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<td>Value</td>
<td>%</td>
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<td>%</td>
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</table>

Source: Barbados Statistical Service
III. Concepts and Tools for Describing Business Cycles

It would appear from the preceding discussion that there is strong evidence of cyclical movements in Barbados and the U.S., with alternating phases of prosperity and recession of irregular duration. Therefore, the use of the classical definition by Burns and Michell (1946) may be appropriate. However, from a purely practical point of view, several difficulties arise when attempting to formalise Burns and Mitchell’s theoretical description and to measure the cycle derived from it. From their description, the idea clearly emerges that the cycle encompasses several expansionary phases, which occur very close together across various spheres of economic activity. Should the cycle therefore be captured utilising a single composite indicator such as GDP or using several variables to represent economic activity? A second problem also arises from their definition, namely, how to identify and measure the cycle, its duration or even its amplitude. Harding and Pagan (2004) clearly detailed the nature of these problems and propose a number of possible solutions.

Despite the drawbacks of employing GDP, especially its tendency to underestimate economic activity due to its failure to account correctly for phenomena such as the environment or informal economic activity, it remains the best overall indicator of essential economic information for a given period. No doubt it is for this reason that the premier institutions, for example, the National Bureau of Economic Research (NBER) and the Organisation for Economic Cooperation and Development (OECD), charged with measuring economic activity around the world, have adopted GDP as a measure of quarterly or annual economic activity and as the indicator of choice for the measurement of cycles. For all the above reasons, real GDP is employed in this paper as the reference cycle for the Barbadian and U.S. economies. Indeed, the advantages of using a single indicator are made clear by Bodart, Kholodilin and Shadman-Mehta (2003) who states that “the use of a single GDP series has an important advantage: it allows avoiding the uncertainty about the precise dates of the business cycle turns arising when multiple reference series are utilised”.

With regard to the practical calculation aspects of the description and measurement of the cycle, the econometric literature provides several analytical techniques that are not always
complementary and may even yield contradictory results if not correctly applied. This comes about because analysts may apply these methods in three different ways: directly on the raw GDP series, $y_t$, on the series $z_{1t}$, which represents the difference between $y_t$ and its permanent component, and on the growth rate of $y_t$, represented by $z_{2t}$. As pointed out by Harding and Pagan (2004), these three alternatives have led to some confusion about the correct terminology to use in relation to the cycle. Moreover, certain econometric studies have incorrectly combined cycle dating algorithms and time series decomposition techniques. In this regard, Harding and Pagan (2004) have justifiably declared that, “it is surprising to see academics quoting NBER cycle statistics and, at the same time, either removing a stochastic trend from series such as GDP through use of filters such as Hodrick-Prescott”.

In light of the above clarifications, this paper examines the Barbadian and U.S. business cycles using the series $y_t$ and by way of the nonparametric Bry and Boschan (1971) algorithm and the parametric Markov Switching Regime framework (see Hamilton (1994)), which are the tools of reference for determining the turning points in economic activity.

IV. Determining the Phase Durations

Dating the turning points of a cycle is a crucial step in the study of economic cycles. Firstly, it influences the content of the information disseminated describing the characteristics of economic cycles, that is, the frequency of turning points, the distinctions between major and minor cycles, the duration of peaks and troughs, the symmetry or asymmetry of these phases, their average duration and their variability, etc. Secondly, it is important in order to correctly undertake comparisons of the cyclical profiles of different countries, especially where the intention is to characterize periods of recession and expansion and their degree of synchronization at the international level. Finally, it contributes to the information which must be provided to policy makers in order for them to make the necessary decisions, that is, to anticipate the effects of imminent turning points, to better appreciate the consequences of recessionary and expansionary periods, etc.
Whereas analyses of this specific dating issue have traditionally been confined to a select few economic research institutions in the U.S., such as the NBER, since the beginning of the 1990s there has been an explosion of studies from institutions all over the world, most notably the OECD on the European Union (see Allard (1994) for France, Bodart, Kholodilin and Shadman–Mehta (2003) for Belgium, and Bruno and Otranto (2003) for Italy)). Today, the various methods utilised in dating and documenting economic cycles can be classified according to two broad categories: nonparametric and parametric methods. Nonparametric models have been criticized for using ad hoc dating rules while parametric models have the inconvenience that all the business cycle analysis depends on the underlying statistical model chosen.

**The Bry and Boschan Non-Parametric Dating Procedure**

The Bry and Boschan (1971) procedure is the most popular method for the selection of turning points. It consists of the *ad hoc* encoding of filters under rules devised by Burns and Mitchell (1946) and was developed in such a way as to reproduce the results of applying the NBER’s dating criteria. It operates on the original data and isolates local minima and maxima in a time series, subject to constraints on both the length and amplitude of expansions and contractions. These constraints are concerned principally with the alternation of peaks and troughs and the persistence of downturns and upturns.

In practice, the Bry and Boschan (1971) procedure consists of six phases of successive application of moving average filters and the treatment of extreme values (see Box below).

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1 Among others, Artis, Kontolemis and Osborn (1997), Harding and Pagan (2002b) and Artis, Marcellino and Proietti (2004) suggest alternative refinements of the Bry – Boschan seminal dating algorithm, based primarily on the lengths of the expansions and recessions and the alternating peaks and troughs.
Outline of the Bry and Boschan Procedure

**Step 1**: Identification and replacement of extreme values.

**Step 2**: Determination of cycles using the standard deviation of the moving average filter. For this and subsequent steps, there are constraints on the alternation of peaks and troughs by selecting the highest of the multiple peaks and the deepest of the multiple troughs.

**Step 3**: Application of a Spencer Curve on the series resulting from Step 2 and updating of the turning points. Elimination of the cycles with the shortest duration.

**Stage 4**: Determination of the turning points in the series resulting from Step 3 by way of a new moving average filter, the order of which must be calculated. Elimination of the cycles with durations that are too short.

**Stage 5**: Determination of the turning points in the original series, taking into consideration the information garnered from Step 4. Elimination of the cycles and phases with durations that are too short.

**Stage 6**: Final selection of turning points.

In essence, this algorithm selects the peaks and troughs that are candidates for turning points and then applies a series of operations in order to eliminate the points that do not satisfy the criteria characterising cycles.

Bruno and Otranto (2003) highlight the need to generalise the Bry and Boschan (1971) procedure within a multivariate framework. They therefore review several solutions proposed in the literature, classifying them into two groups: the indirect approaches, which aggregate the turning points identified using several different series, and the direct approaches, which construct a composite indicator based on different economic variables and which apply the Bry and Boschan procedure to identify the turning points in activity. The latter is the approach taken in the present study.

**The Hamilton Parametric Dating Procedure**

Contrary to the Bry-Boschan method and similar approaches, parametric models assume a statistical model and use it to deduce the chronology from the turning points and the characteristics of the cycle. Stemming from the postulate that the description of the business cycle brings into play a finite number of possible situations – normally the two phases of
recession and expansion – some authors, most notably, Hamilton (1994), have used Markov chains to represent the economic evolutions\(^2\). The Hamilton approach complements or is sometimes, even preferable to the Bry and Boschan procedure. In fact, comparing these two approaches allows one to highlight the following aspects:

- The magnitude of growth rates needed to trigger a regime-shift in the Hamilton model will change from state to state, whereas it remain constant across states in the Bry and Boschan algorithm (Harding and Pagan (2002b), Owyang, Piger and Wall (2003)).
- The Hamilton approach is more precise in terms of the mathematical proofs for the identification of turning points and;
- Due to the fact that it is based on an econometrically estimated specification, the Markovian approach clearly allows for statistical inference as well as the forecasting of turning points (see Bodart, Kholodilin and Shadman-Mehta (2003)).

When considering the variable \(\Delta y_t\), whose successive stages describing an economic situation are essentially positioned on either the ascending or descending trajectory, Hamilton (1994) proposed the representation of its evolution by an autoregressive model that includes a two-stage Markov chain. The keystone of such a regime-switching model is precisely the explicit description and probabilistic evaluation of the passage from one stage to another. With two stages, a relatively simple parameterisation within this category of models is obtained a priori:

\[
\begin{align*}
\Delta y_t &= \mu_y + \sum_{i=1}^{r} \delta_i \Delta y_{t-i} + \epsilon_t \\
\mu_y &= \mu_0 (I - S_i) + \mu_1 S_i \\
Pr[S_t = 1|S_{t-1} = 1] &= p_{11}, \text{ and } Pr[S_t = 0|S_{t-1} = 0] = p_{00}
\end{align*}
\]

\(^2\) Other alternatives that have been employed to distinguish the different phases of the business cycles include the threshold autoregressive process of Tsay (1989) and the smooth transition autoregressive models of Teräsvirta (1994). For an useful survey see Camacho and Perez-Quiros (2002).
with \( \varepsilon_t \sim i.i.d. N(0, \sigma^2) \) and \( S_t \), a stage variable defined by \( S_t = \{0,1\} \). The first two equations describe the trajectory of \( \Delta y_t \) by introducing regime changes in terms of both the levels and variance. Because of the non-observability of \( S_t \), the estimation of this model cannot be envisaged in such a way that the stochastic process that generates the values of \( S_t \) is restricted. In probabilistic terms, this property of short memory is expressed as:

\[
P\{S_t = j|S_{t-1} = i, S_{t-2} = k \ldots \} = P\{S_t = j|S_{t-1} = i, I_{t-1} \} = p_{ij} \quad \forall i, j \in \{0,1\}
\]

Furthermore, in accordance with the value of \( r \), the model brings into play dynamic effects, which are more or less complex. However, the more autoregressive terms of \( \Delta y_t \) that are incorporated, the larger the number of sequences of stages which describe the evolution of the increasing economic activity. This represents a serious stumbling block to the estimation of the parameters in question. Consequently, it is then necessary to find a compromise around in optimal value of \( r \).

The parameters to be estimated with a clear economic interpretation are the following: the ergodic probabilities \((p_{00} \text{ and } p_{11})\) of the transition matrix \( P = (p_{ij}) \); the means associated with each regime \((\mu_0 \text{ when the series } \Delta y_t \text{ is in a state of recession and } \mu_1 \text{ when it is in a state of expansion})\) and; the variance \( \sigma^2 \).

In order to estimate these parameters, Hamilton proceeded using maximum likelihood, based on the expectancy-maximisation (EM) algorithm, as well as combining this with the non-linear optimisation procedure of Broyden, Fletcher, Goldfarb and Shanno (BFGS) (see Press et al (1989)).

The EM algorithm, one of the techniques generally employed for the estimation of likelihood functions in stochastic models with hidden variables, looks for the vector \( \theta = (\mu_0^*, \mu_1^*, p_{00}^*, p_{11}^*, \sigma^*, \phi_1^*, \phi_2^*, \phi_3^*, \phi_4^*) \) by using the angle of the iterative procedure. From an initial condition \( \theta_0 \), it determines the successive estimators \((\theta_i)_{i=1,2,...} \) until it converges at the solution \( \theta^* \). During the iteration \( i \), two stages of calculations are carried out: stage E
(expectation) computes the interaction of parameters $\theta$, the value of the likelihood function associated with the observations and the Markovian states $S$; the stage M (maximization) looks for the new interplay between the parameters $\theta_{i+1}$ which maximize the previously estimated likelihood.

The BFGS procedure takes place in order to allow the implementation of the passage from the point $\theta_i$ to the point $\theta_{i+1}$. As with many optimization methods, it is based on the logic of movement from $\theta_i$ to $\theta_{i+1}$ following a direction $d_i$ and a progression step $t_i$. The iterative process is therefore based on the recurrence of the form $\theta_{i+1} = \theta_i + t_i \cdot d_i$ where $d_i$ and $t_i$ can be calculated using various methods. For the Newton-Raphson method, the BFGS algorithm proposes two choices:

- $d_i$ is evaluated by pre-multiplying the gradient $g$ by the matrix $G$, where $G$ starts with a diagonal matrix. At each iteration, it is updated based on the change in parameters and on the gradient, in an attempt to determine the curvature of the function. The basic theoretical result governing this is that if the function is truly quadratic, and if exact line searches are used, then in $n$ iterations, $G$ will be equal to $-H^{-1}$. If the function isn’t quadratic, $G$ will be an approximation to $-H^{-1}$, $H$ being the Hessian matrix (Doan (2002)).

- the value of the rate of change is selected by the so-called line search procedures.
V. Results from the Bry and Boschan Procedure

The Barbadian Reference Cycle

While the Bry and Boschan algorithm was initially created for monthly series, with specific parameters imposing the sequence of peaks and troughs, as well as the duration and amplitude of the different phases of the cycle, it has subsequently been adapted for use with quarterly data. This paper uses a slightly modified version of the RATS program written by Bruno and Otranto\(^3\), which itself is a translation of the GAUSS code written by Harding and Pagan (2001). The set of parameters \(K=L=2\) commonly adopted for quarterly data as well as the Spencer moving average of order 4 were used, replacing the parameters \(K=L=6\) and the Spencer moving average of order 15. Therefore, a turning point \(y_t\) corresponds to a local maximum or minimum of more or less two quarters: \(y_t\) is a trough if and only if \((\Delta_2 y_t, \Delta y_t) < 0\) and \((\Delta y_{t+1}, \Delta_2 y_{t+1}) > 0\); \(y_t\) is a peak if and only if \((\Delta_2 y_t, \Delta y_t) > 0\) and \((\Delta y_{t+1}, \Delta_2 y_{t+1}) < 0\), with \(\Delta_2 y_t = y_t - y_{t-2}\) and \(\Delta y_t = y_t - y_{t-1}\).

The turning points identified are shown in Figure 3 over the thirty-year period. It should be noted that Barbados’ real GDP registered only four troughs – 1975:1, 1982:3, 1992:3 and 2002:1 – and, therefore, a similarly small number of peaks, in 1981:3, 1988:4 and 2000:2. Tables 4 and 5 reproduce the durations of the different phases of the cycles: the complete cycles from trough to trough and peak to peak, the expansionary phases between trough and peak and the recessionary phases covering the periods from peak to trough.

\(^3\) Many thanks to G. Bruno and E. Otranto for the use of their RATS program.
Figure 3: Barbadian Real GDP (Logarithms), Bry-Boschan Reference Cycle Dates
A notable feature of these measurements is the strong asymmetry between phases, with the expansionary phases generally being much longer in duration than the recessionary phases. The expansionary phases consist of between 24 and 30 quarters, whereas the recessionary phases, which are much more variable, lasted between 3 and 14 quarters.

Table 4: Durations of the Phases of the Barbadian Real GDP Cycle (Quarterly)

<table>
<thead>
<tr>
<th>Expansions</th>
<th>Period</th>
<th>Duration in Quarters</th>
<th>Contractions</th>
<th>Period</th>
<th>Duration in Quarters</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Descriptive Characteristics of the Phases of the Barbadian Business Using the Bry and Boschan Approach Cycle

<table>
<thead>
<tr>
<th></th>
<th>Expansion</th>
<th>Contraction</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average duration</td>
<td>29.7</td>
<td>10.3</td>
<td>36.3</td>
</tr>
<tr>
<td>Median duration</td>
<td>26</td>
<td>6</td>
<td>38</td>
</tr>
<tr>
<td>Max duration</td>
<td>30</td>
<td>14</td>
<td>40</td>
</tr>
<tr>
<td>Min duration</td>
<td>24</td>
<td>3</td>
<td>31</td>
</tr>
<tr>
<td>Proportion of time</td>
<td>74.16%</td>
<td>25.84%</td>
<td>100</td>
</tr>
<tr>
<td>Ratio expansion/contraction</td>
<td>2.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average amplitude</td>
<td>28.49%</td>
<td>-12.18%</td>
<td></td>
</tr>
<tr>
<td>Steepness</td>
<td>0.96</td>
<td>-1.18</td>
<td></td>
</tr>
</tbody>
</table>

These empirical results are notable when compared with previous results for both developed and developing countries. Firstly, this chronology reveals that the three periods of expansion each lasted at least six years and occurred in the 1970s, 1980s and 1990s, respectively. This observation is consistent with the information in Figure 1 of Section II above. In effect, with respect to the phases in real GDP, growth fluctuated significantly, however, the contractionary phases were not sufficiently significant to be considered real recessionary phases.
In conjunction with these broadly descriptive measures of the duration of the phases of a cycle, other measures are just as widely utilised to capture the evolution of cyclical movement. For the most part, they take into account the characteristics of the average duration, range and symmetry, which have been discussed by Harding and Pagan (2001) among others, and also allow for international comparisons.

It is also important to highlight the fact that the characteristics of the Barbadian business cycle are somewhat different from those of other developing countries. Considering, for example, a varied group of countries in Africa (Côte d’Ivoire, Malawi, Nigeria, South Africa and Zimbabwe), South America (Chile, Colombia, Mexico, Peru and Uruguay), as well as Asia and North Africa (India, South Korea, Malaysia, Morocco and Pakistan), Rand and Tarp (2003), using the Bry and Boschan procedure, showed that the average duration of their cycles was between 7 and 18 quarters. Conversely, the characteristics of the Barbadian cycle appear to be more similar to those of developed countries, like Australia (see Harding and Pagan (2002b)) and France and Spain (see Harding and Pagan (2001)). In fact, the main difference that appears between the Barbados cycle and the business cycles of developed countries relate to the characteristics of the contraction phase where Barbados stands out as having longer contractions with more pronounced amplitude.

**The U.S. Business Cycle**

The indicator dates and characteristics of the U.S. economic cycle are officially assessed by the NBER. For the period concerned in this study, Table 6 presents a reproduction of the results found on their website, [http://www.nber.org/cycles.html](http://www.nber.org/cycles.html). It should be noted that the NBER’s definition of a recession differs slightly from the one used by Harding and Pagan (2001) in their Bry and Boschan quarterly procedures: “The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP. Rather, a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production and wholesale-retail sales”. Furthermore, contrary to the Bry and Boschan approach, which only selects periods of negative growth as periods of recession, the NBER’s dating method is less restrictive and take the most recurring events into consideration, when defining a recession.
Notwithstanding the above differences in methodologies, the results provided by the RATS procedure for Barbados above are practically the same as for the U.S. cycle. Analysing the U.S. cycle in Table 6 and comparing the chronologies of Barbados and the U.S. cycles, the following can be observed:


- since the early 1970s, economic change in the U.S. has been characterized by three long periods of growth (1975:1-1980:1 for 20 quarters, 1982:4-1990:3 for 31 quarters and 1991:1-2001:4 for 40 quarters) and the 1980s were more volatile than either of the preceding or following decades. In comparison, the Barbadian economy also experienced three similar long periods of growth (1975:1-1981:3 for 26 quarters, 1982:4-1988:4 for 24 quarters and 1992:4-2000:2 for 30 quarters) and a volatile 1980s phase, reflecting a long period of structural transformation marked by an increase in productivity gains.

### Table 6: NBER Quarterly Chronology of Business Cycles in the U.S.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Trough</th>
<th>Expansion (previous trough to this peak)</th>
<th>Contraction (peak to trough)</th>
<th>Cycle (trough from previous trough)</th>
<th>Cycle (peak from previous peak)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980:1</td>
<td>1975:1</td>
<td>20</td>
<td>2</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>1981:3</td>
<td>1980:3</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>1990:3</td>
<td>1982:4</td>
<td>31</td>
<td>2</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>2001:1</td>
<td>1991:1</td>
<td>40</td>
<td>3</td>
<td>43</td>
<td>42</td>
</tr>
</tbody>
</table>

Source: [http://www.nber.org/cycles.html](http://www.nber.org/cycles.html)
VI. Results from the Hamilton Procedure

The Barbadian Reference Cycle

With the help of the RATS programme, the estimators of the parameters of the model are calculated using the first differences of the real GDP series for the period 1975:2 – 2003:4. The findings presented in Table 7 have been obtained after 25 iterations. A notable feature of the results is that all of the parameters are significant. The average values of the GDP growth rates during the expansion and recession phases are estimated at 0.918% and −1.225%, respectively. With these two values, whose statistical significance is strong, the model allows for clear differentiation of the dynamics associated with the two stages. It can also be noted that the probability of staying in recession $p_{00}$ is relatively high (0.835) as well as that of staying in expansion $p_{11}$ (0.955). From these two probabilities, the average duration of the corresponding regimes can be deduced and this process suggests that the recession phase lasts for an average of 6 terms ($1/(1 - p_{00})$), while the expansionary phase takes for an average of 22 terms ($1/(1 - p_{11})$). The results are lower than the 10 and 30 terms calculated by the Bry and Bryson dating procedure.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Barbados Estimates (*)</th>
<th>U.S. Estimates (*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.918 (7.638)</td>
<td>0.937 (10.253)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-1.225 (-4.015)</td>
<td>-1.328 (-5.345)</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.955 (33.186)</td>
<td>0.963 (53.127)</td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.835 (9.673)</td>
<td>0.394 (2.034)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.937 (13.712)</td>
<td>0.576 (15.036)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>-0.300 (-3.007)</td>
<td>0.407 (4.404)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.262 (-2.623)</td>
<td>0.298 (2.983)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-0.339 (-3.351)</td>
<td>-0.229 (-2.270)</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>-0.228 (-2.283)</td>
<td>-0.082 (-0.905)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-252,462</td>
<td>-120,755</td>
</tr>
</tbody>
</table>

(*) : The t-statistics are indicated in parentheses.
**The U.S. Business Cycle**

For the U.S., all the parameters, except $\phi_4$, are significant. The average values of the GDP growth rates during expansions and recessions are estimated at 0.937% and −1.328%. The probability of staying in expansion is fairly high (0.963) but low for staying in a recession (0.394). This latter result contrasts with Barbados whose probability of staying in a recession was 0.835 (see also Figure 4). The average duration of the recession phase is about 2 quarters while the expansionary phase lasts around 27 quarters, which is more in line with the previous results for Barbados.

![Figure 4: Hamilton Recession Probabilities](image)

V. A Statistical Examination of the Synchronization of the U.S. and Barbadian Economic Cycles

A statistical estimation of the actual degree of resemblance between the U.S. and Barbadian economic cycles can be achieved by using the binary variable $S_t$, discussed in the Hamilton (1994) procedure above, which shows the state of the economy. In fact, Harding and Pagan
(2002b) proposed a compound index, \( I_{xy} \), which allows for a calculation of the number of periods during which two countries, \( x \) and \( y \), show parallelisms in their cycles:

\[
I_{xy} = \frac{1}{T} \sum_{t=1}^{T} \left\{ I(S_{xt} = 1, S_{yt} = 1) \right\} + \left\{ I(S_{xt} = 0, S_{yt} = 0) \right\}
\]

where \( S_{xt} \) represents the variable showing the state of country \( x \), \( S_{yt} \) that of country \( y \), \( T \), the size of the sample and \( I \), the indicative function.

The estimated value of 0.83 reveals that there is significant synchronization of the different phases of the Barbadian and U.S. business cycles. For 83% of the total time concerned in the period 1974:Q1-2003:Q4, both cycles were in the same state. This synchronization reflects the strong influence exerted by the U.S. economy on the Barbadian economy.

Another way to examine the degree of business cycle synchronization between two countries at a specific time is to use cross correlation of the cyclical components, that is,

\[
\frac{\text{cov}(y^c_t, x^c_t)}{\left(\text{var}(y^c_t)^*\text{var}(x^c_t)\right)^{1/2}}
\]

where the cyclical component, indexed by \( c \), is determined by the Hodrick-Prescod filter in EVIEWS 5 (Table 8).

<table>
<thead>
<tr>
<th>Delay</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr.</td>
<td>0.19993</td>
<td>0.30561</td>
<td>0.35298</td>
<td>0.35042</td>
<td>0.38822</td>
<td>0.45981</td>
<td>0.53473</td>
<td>0.53850</td>
<td>0.49344</td>
</tr>
</tbody>
</table>

The results indicate that there are delays of one or two quarters in the Barbadian cycle, relative to the U.S. cycle. The coefficients support the fact that these two economies vary in the same way. More precisely, the growth or recession in the U.S.’s economy is followed by a similar variation in the Barbadian economy.
Finally, a formal test of asymmetry is done using the following formula (see Calderón, Chong and Stein (2002)):

\[
\text{asymm} \left( Y_{US}, Y_{BDOS} \right) = \sigma \left( \frac{Y_{US \tau \tau} - Y_{US \tau \tau-1}}{Y_{BDOS \tau \tau} - Y_{BDOS \tau \tau-1}} \right)
\]

where \( \sigma(\cdot) \) represents the standard deviation computed over \( \tau \) periods and \( Y \) represents output in logs. If \( \text{asymm} \left( Y_{US}, Y_{BDOS} \right) = 0 \), both countries have analogous cycles. Using the values for Barbados and the US GDPs, \( \text{asymm} (\cdot) \) turns out to be 0.007, implying similarities in the business cycles of the two countries.

**Conclusion**

In the context of an international environment characterised by the increasing convergence of national economies, subject to the constraints of dwindling budgetary receipts, policy makers have had to place greater emphasis on the formulation and evaluation of public policy. Consequently, economic analysts are often asked to examine the evolution of economic variables in an attempt to anticipate the occurrence of strong upturns or downturns in economic activity.

In the face of such expectations, this paper has presented empirical evidence from both nonparametric and parametric methods on the chronology of the Barbadian economic cycle over the last three decades and have shown that it is closely linked to that of the U.S. business cycle. This close association between the two countries’ cycles reflect their historical and current trading arrangements. From a policy point of view, a chronology for Barbados provides a framework for pinpointing the position of the Barbadian economy in its current cycle. Moreover, it can add useful information in the development of leading and coincident indicator indices. Furthermore, that there exists a close relationship between the U.S. and Barbados, with a lag of about six months, signifies that the Barbadian growth experience cannot be undertaken without analyzing the dynamic relationship of the two countries.
References


