A Contingent Claims Approach to Measuring Insolvency Risk:
An empirical assessment of the impact of the global financial crisis on
Jamaica and its financial sector

Jide Lewis\textsuperscript{1,2}

Abstract

This paper proposes to compute a probability of default measure for Jamaica and its financial system using the contingent claims approach (CCA). The contingent claim approach is based on Black-Scholes-Merton's option pricing theory, where an entity’s equity can be viewed as a call option on the value of its assets. Estimates for the distance-to-default and the probability of default for the sovereign and publicly listed financial institutions in the bank and non-bank sector in Jamaica are presented between 2005 and 2010. The resulting vulnerability indicators are then used to retroactively to assess the impact of the global financial crisis on the sustainability of Jamaica’s debt profile and the viability of its financial sector. The results underscore the framework’s ability to act as an early warning indicator of macro-financial vulnerabilities and highlight possibility of contagion between various sectors of the Jamaican economy.

Keywords: Contingent claims approach, Credit risk, Financial Stability, Early Warning Model.

JEL Classification: G13, G32, E4, G2.

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\textsuperscript{2} The views expressed in this paper are those of the author and do not necessarily reflect the views of either the Bank of Jamaica or the University of the West Indies.
I. Introduction

The use of option-pricing models in bankruptcy prediction provides guidance about the theoretical determinants of bankruptcy risk and supply the necessary structure to extract bankruptcy–related information from prices derived from the equity, foreign exchange and bond markets. These markets provide a potentially superior source of information regarding the risk of insolvency because it aggregates high frequency information from several markets simultaneously about the collective views of many investors. That is, valuing assets using marked-to-market prices and incorporating contingent liabilities provides a more nuanced assessment of the inherent risks within the balance sheet of either a private firm or a sovereign nation. It must be noted that the market value of assets of a corporation, financial institution, or sovereign cannot be observed directly. However, given the observed prices and volatilities of market traded securities, one can estimate the implied value and volatilities of the underlying assets (Merton and Bodie 1995). Option-pricing models prove to be an effective methodology whereby the information from these markets can be extracted to deduce forward looking estimates of the probability of bankruptcy over a specified time horizon.

Based on the approach by Black and Scholes (1973) and Merton’s (1974) extension of the Black-Scholes model, the firms’ equity can be viewed as a call option on the value of an entity’s assets. When the value of the assets falls below the face value of the liabilities, the call option is left unexercised, and the bankrupt entity is turned over to its debt holders. Once the Black Scholes and Merton (BSM) model is employed to estimate the marked-to-market value of an entity’s assets as well as its volatility then a set of risk indicators can be formulated to serve as a gauge of the likelihood of bankruptcy of the entity. Contingent claims analysis (CCA) then is the application of the Black-Scholes-Merton model to a wide range of entities including corporates, financial institutions and sovereign nations.

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3 An implied value refers to an estimate derived from other observed data. Technique for using implied values are widely practiced in option pricing and financial engineering applications.
The framework can be used to understand many types of crises and risk transfers between various sectors of an economy that cannot be easily analyzed with other techniques. The framework can, for example, help identify situations where volatility in one sector gets magnified and negative feedback loops then trigger severe crises in other sectors of the economy. This risk-transmission process is a function of the linkages in the capital structure between various sectors as well as the correlations between asset prices across various sectors of an economy.\textsuperscript{4,5} For example, financial distress in the banking sector can be transmitted to the government by an increase in the value of the implicit guarantees the government provides to the financial sector. In the case of Jamaica, this implicit guarantee became explicit after the financial system distress in the 1990s which has been estimated at 40.0 per cent of Gross Domestic Product (GDP).

On the other hand, government's financial distress or defaults can transmit risk to the financial system. This is particularly true where the bank and non-bank sectors hold a significant proportion of government securities within their investment portfolios. In this case, a negative shock to the government's financial position can have a detrimental impact on the viability of these financial institutions. A vicious cycle could then arise, when the lower value of government securities in turn lowers the value of bank assets, and raise the implicit financial guarantee, which in turn lowers government assets further.\textsuperscript{6}

Finally, the inability of a government to sustainably finance its fiscal accounts and its contingent liabilities can cause distress for the government which can transmit risk to external holders of government debt. Higher spreads demanded by the debt holders to cover the credit risk in government debt could lead to higher borrowing costs on government debt which could lead to depreciation in the exchange rate and the resulting feedback could potentially further worsen the sovereign's financial position.

\textsuperscript{5} Banking sector distress arising from a significant increase in non-performing loans, a deposit run and precipitous decline in the value of assets can result in a large increase in the government's implicit guarantee.
The organization of the paper is as follows. The next section, presents a brief survey of the bankruptcy/credit risk literature. Section III presents the Merton model for bankruptcy risk. Section IV contains the results, which presents two metrics for bankruptcy risk, namely distance-to-default and probability of default for Jamaica as well as deposit taking institutions (DTIs) and non-bank financial institutions (NBFIs) in Jamaica between 2005 and June 2010. This period gives us an opportunity to evaluate the solvency of the government and financial sector throughout the recent global financial distress. The paper concludes in Section V with key policy implications.

II. Literature Review

Some of the earliest works in the area of insolvency risk used balance sheet data to derive probability of default metrics (see for example Altman (1968) and Ohlson (1980). The Altman's z-score, for example, is a linear discriminant model where borrowers are classified into either high or low default risk categories. Although the framework does not directly give a probability of default, the results can be mapped to a credit rating system which would in turn yield the desired estimates. The framework uses five fundamental balance sheet ratios: working capital, retained earnings, earnings before interest and taxes and sales to total assets as well as the ratio of the market value of equity to the book value of total liabilities. Similar to Altman, Ohlson (1980) uses linear discriminant analysis to derive the probability of default of a firm. The framework proposed by Ohlson (1980) uses nine balance sheet ratios and utilized the maximum likelihood techniques to estimate a logit model of the probability of default for various firms.

There are several reasons why one may not want to use accounting based methods to estimate the probability of insolvency. While the probability estimates are statements about future events, financial statements are designed to measure past performance, and

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7 The discriminant analysis methodology identifies linear combinations of features which characterize or separate two or more classes of objects or events.
8 Some practitioners utilized a logistic function to convert the z-scores derives from the discriminant analysis into a probability function.
thus, may not be very informative about the future status of the firm. Financial statements are formulated under the going-concern principle, which assumes that firms will not go bankrupt. Thus, their ability to accurately and reliably assess the probability of bankruptcy will be limited by design. Additionally, the conservatism principle used in accounting often causes asset values to be understated relative to their market values. This is particularly true for fixed assets and intangibles. Downward-biased asset valuations will cause accounting-based leverage measures to be overstated. These aspects of the accounting system will limit the performance of any accounting-based insolvency measure. Another important deficiency in most accounting based insolvency measures is their failure to incorporate a measure of asset volatility. Volatility is a crucial variable in bankruptcy prediction because it is a key component in computing the likelihood that the firm will be unable to repay its debts within a specific horizon. All things being equal, the probability of bankruptcy is increasing with volatility and therefore two firms with identical financial ratios can have substantially different credit risk depending on their asset volatilities (Hillegeist, et al. 2003).

In contrast to the balance sheet approach, the Merton model is a structural model of bank insolvency, which can be used to derive the probability of default for an entity. The probability of default of a firm is captured as an endogenous process and is a measure of the likelihood that a firm's assets in the future are likely to fall below its liabilities rendering the entity bankrupt. The Merton model and the contingent claims approach (CCA) which extends the framework to assess multiple sectors is based on three principles: (i) the value of liabilities flows from assets; (ii) liabilities have different seniority; and (iii) there is a random element to the way an asset’s value evolves over time.

The main advantage of the CCA/Merton model is that it uses observable balance sheet and financial market data along with volatility to construct a measure of default risk. The

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9 The CCA is used by (i) major credit rating agencies to monitor and assign credit ratings, (ii) financial institutions to inform interest rate pricing on loans and set adequate levels of regulatory capital, and (iii) investment banks and insurance companies to assess value-at-risk. For example, when applied across a portfolio of firms, the probability of default multiplied by weighting within the portfolio creates a value –
ability to translate continuously adjusting financial market price information into current market value estimates of asset value is especially important given the speed with which economic conditions change relative to the time span between releases of consolidated – accounting balance sheet information. Furthermore, balance sheet information arrives with a significant lag, usually 90 days after the quarter or annual publication date. The CCA combines the capital structure of the balance sheet with current price information from financial markets to construct a market value estimate of the current balance sheet along with forward looking indicators of vulnerability. In addition, the CCA distinguishes itself from other vulnerability analysis by recognizing the important role of volatility in determining default probabilities. Increases in volatility increase the option value and benefits of equity holders at the expense of bondholders. By capturing volatility, the CCA accounts for the fact that firms with the same capital structures may have different distance-to-distress and default probabilities. The CCA methodology also incorporates nonlinearities which yield significant improvements over traditional linear relationships in vulnerability analysis. In option pricing theory, the value of the option is dependent on changes in the underlying asset. The nonlinearity of the Black-Scholes methodology allows for a more accurate description of changes in vulnerabilities arising from large changes in asset prices. Linear relationships, on the other hand, may fail to be adequate indicators for surveillance purposes as they may underestimate the evolution of risk over time as result of exogenous shocks (see Gray, Merton and Bodie, 2003).

These potential benefits, however, come at the cost of relating on the models’ simplifying assumptions, many of which do not hold in practice. These assumptions can introduce errors and biases into the resulting insolvency estimates. For example, most estimates of probability implicitly assume that all of the entity’s liabilities mature in one year. For most firms, this substantially underestimates the actual duration of the liabilities and can lead to higher insolvency estimates. The framework also assumes that if the value of the firm’s assets is less that its total liabilities at time $T$, then the firms simultaneously

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at-risk (VaR) indicator that is then used in conjunction with other VaR indicators to adjust capital adequacy or allow the firm to offset risk exposure by entering into offsetting financial transactions.

10 The Merton model is easily modified to compute the probability of bankruptcy over any time horizon by changing the time parameter $T$. 
defaults, declares bankruptcy and causelessly turns control over to the bondholders. In practice, bankruptcy does not always occur when this economic condition is met. Frictions in the bankruptcy process, such as violations of strict priority and deadweight costs, can lead to forced debt re-negotiations that do not entail a formal bankruptcy filing. The entity can also avoid an immediate bankruptcy filing by meeting its current obligations if some of its liabilities are due at a later date. On the other hand, some firms will file for bankruptcy even when they are economically solvent. Firms will strategically enter into bankruptcy ‘early’ to break unfavorable contracts and protect themselves from litigation. Additionally, short-term liquidity constraints can prevent the firm from meeting its obligations even though its total liabilities are less than the market value of its assets. To the extent that these situations can and do occur, the empirical performance of the Merton model will be reduced because these possibilities are not incorporated into the framework. Also, many firms that might otherwise declare bankruptcy are either acquired or liquidated outside of the bankruptcy process. Finally, the option-based approach is predicated on the assumption that the stock market, foreign exchange and bond markets impound all publicly-available information about future prospects of insolvency into prices. This, however, may not hold in practice. In particular, prior studies suggest that the market does not always accurately reflect all the information in financial statements. Ultimately, whether the probability of bankruptcy is derived from an option-pricing model or an accounting–based insolvency measure is an empirical question.11

III. Data

Government of Jamaica (GOJ) Data

The GOJ data is constructed using daily data from the Bank of Jamaica (BOJ) on the monetary base between December 2004 and June 2010. Daily foreign exchange rates, available from the BOJ, between the Jamaica and the United States (US) are used to convert the monetary base into US dollar equivalents. Quarterly balances for the domestic debt stock as well as short and long-term foreign denominated debt were also

1111 This entire section is a summary of arguments made by Hillegeist et al. (2003).
collated for the period under review. Short-term foreign currency debt was classified as instruments with a maturity of less than five years and long-term foreign currency debt had maturity profiles in excess of five years. All debt related data were collected from the Ministry of Finance (MOF) and converted (where necessary) into US dollar equivalents.

Financial Sector Data

The data set for the financial sector was constructed using balance sheet information from the Jamaica Stock Exchange (JSE) and the BOJ as well as stock market related data available from the JSE. The banking category consists of four highly liquid stocks listed on the JSE. These institutions are National Commercial Bank (NCB), Bank of Nova Scotia (BNS), First Caribbean International Bank (FCIB) and Capital and Credit Merchant Bank (CCMB). For the non-bank category, the five firms chosen were Mayberry (MBL), Pan Caribbean Financial Service (PANCAB), Jamaica Money Market Brokers (JMMB) and Life of Jamaica (LOJ). These firms represent some of the largest financial institutions in Jamaica and their insolvency risk has a direct and significant influence on the health of the Jamaican financial system. The sample data covers the period from December 2004 to June 2010.

The analysis uses daily stock price data between end-December 2004 and 30 June 2010. Monthly data on the shares outstanding was gathered from hard-copy data available from the JSE. Both these series are used to compute the market value of each institution’s equity as well as the volatility of the institution’s equity. The historical volatility of equity is measured by taking the standard deviation of the returns on the equity valuations for traded securities over two-hundred and fifty trading days where returns are computed as the log of the ratio of value of equity at time \( t \) and closing value of equity at time \( t-1 \). Daily volatility of equity returns is annualized by multiplying by the square root of two-

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12 Information on Shares Outstanding was only available hard-copy back do December 2005
13 Data for Royal Bank of Trinidad and Tobago (RBTT) and Guardian Holding Limited were omitted from the banking group and non-bank group, respectively as they were found to be relatively illiquid over the period of the analysis.
hundred and fifty trading days. Bank's balance sheet information available quarterly from the BOJ is used to gather information on the current liabilities and long-term liabilities which is used to calculate the default barrier (DB). Balance sheet items including Deposits, Due to BOJ, Commercial Banks, Specialized Institutions, Other Specialized Institutions and Securities sold under Repo, and Other Current Liabilities are used to compute the short-term liabilities. Other Liabilities on the balance sheet are used to compute long-term liabilities. For the NBFIs annual balance sheet data available from filings made to the JSE by participants on the exchange are used to derive figures for both current and long-term liabilities used to compute the DB. The descriptive statistics for the bank and non-bank specific variable are shown in Table 1 and Table 2, respectively.

Table 1. Descriptive Statistics for Deposit-Taking Institutions Included in Sample, 2004 Q4 to 2010 Q2

<table>
<thead>
<tr>
<th>Stats</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Volatility (%)</td>
<td>21.6</td>
<td>20.0</td>
<td>16.5</td>
<td>13.6</td>
<td>20.1</td>
<td>28.5</td>
<td>26.8</td>
</tr>
<tr>
<td>Current Liabilities (E.O.P)</td>
<td>109.8</td>
<td>110.0</td>
<td>109.3</td>
<td>111.8</td>
<td>115.8</td>
<td>119.4</td>
<td>128.8</td>
</tr>
<tr>
<td>LT Liabilities (E.O.P)</td>
<td>1.8</td>
<td>2.1</td>
<td>2.4</td>
<td>1.7</td>
<td>1.6</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Mean Value of Equity</td>
<td>56.9</td>
<td>48.3</td>
<td>43.0</td>
<td>52.9</td>
<td>54.4</td>
<td>34.4</td>
<td>40.7</td>
</tr>
<tr>
<td>BNS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Volatility (%)</td>
<td>22.48</td>
<td>45.84</td>
<td>32.84</td>
<td>12.90</td>
<td>17.71</td>
<td>22.37</td>
<td>23.05</td>
</tr>
<tr>
<td>Current Liabilities (E.O.P)</td>
<td>111.8</td>
<td>110.8</td>
<td>108.0</td>
<td>111.4</td>
<td>117.3</td>
<td>125.3</td>
<td>122.8</td>
</tr>
<tr>
<td>LT Liabilities (E.O.P)</td>
<td>1.7</td>
<td>1.7</td>
<td>2.6</td>
<td>2.0</td>
<td>1.9</td>
<td>1.9</td>
<td>2.1</td>
</tr>
<tr>
<td>Mean Value of Equity</td>
<td>67.5</td>
<td>61.9</td>
<td>61.5</td>
<td>70.4</td>
<td>69.2</td>
<td>53.4</td>
<td>66.7</td>
</tr>
<tr>
<td>FCIBJ</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Current Liabilities (E.O.P)</td>
<td>18.1</td>
<td>18.3</td>
<td>19.1</td>
<td>20.9</td>
<td>21.6</td>
<td>22.3</td>
<td>24.9</td>
</tr>
<tr>
<td>LT Liabilities (E.O.P)</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Mean Value of Equity</td>
<td>4.0</td>
<td>3.3</td>
<td>4.6</td>
<td>6.2</td>
<td>6.3</td>
<td>4.3</td>
<td>3.5</td>
</tr>
<tr>
<td>CCMB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Volatility (%)</td>
<td>24.33</td>
<td>22.77</td>
<td>24.51</td>
<td>26.22</td>
<td>34.35</td>
<td>40.91</td>
<td>41.61</td>
</tr>
<tr>
<td>Current Liabilities (E.O.P)</td>
<td>35.4</td>
<td>35.9</td>
<td>29.3</td>
<td>24.2</td>
<td>28.5</td>
<td>30.4</td>
<td>31.8</td>
</tr>
<tr>
<td>LT Liabilities (E.O.P)</td>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Mean Value of Equity</td>
<td>11.0</td>
<td>14.9</td>
<td>9.6</td>
<td>6.7</td>
<td>7.0</td>
<td>3.5</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Notes: Unless otherwise stated all units are in Jamaica Dollar billions. Data for 2010 covers the period January 2010 to June 2010.
Table 2. Descriptive Statistics for Non-Bank Financial Institutions Included in Sample, 2004 Q4 to 2010 Q2

<table>
<thead>
<tr>
<th>Stats</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>JMMB Max Volatility</td>
<td>21.9</td>
<td>21.7</td>
<td>22.4</td>
<td>22.5</td>
<td>23.1</td>
<td>28.6</td>
<td>20.4</td>
</tr>
<tr>
<td>Current Liabilities (E.O.P)</td>
<td>57.3</td>
<td>57.7</td>
<td>74.2</td>
<td>82.0</td>
<td>93.2</td>
<td>93.2</td>
<td>100.2</td>
</tr>
<tr>
<td>LT Liabilities (E.O.P)</td>
<td>0.0</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>2.6</td>
<td>2.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Mean Value of Equity</td>
<td>23.4</td>
<td>22.5</td>
<td>18.8</td>
<td>15.4</td>
<td>14.9</td>
<td>6.9</td>
<td>5.9</td>
</tr>
</tbody>
</table>

| MAYB Max Volatility          | -    | -    | 28.6 | 26.4 | 33.8 | 36.0 |
| Current Liabilities (E.O.P)  | 12.7 | 12.9 | 18.0 | 17.8 | 19.4 | 19.4 | 9.4  |
| LT Liabilities (E.O.P)       | 2.8  | 1.9  | 1.1  | 2.7  | 2.2  | 2.2  | 9.4  |
| Mean Value of Equity         | -    | 3.8  | 2.9  | 4.9  | 3.9  | 2.3  | 2.9  |

| PANCAB Max Volatility        | 34.8 | 28.0 | 25.5 | 24.6 | 23.6 | 24.1 | 23.4 |
| Current Liabilities (E.O.P)  | 34.3 | 33.5 | 37.2 | 42.1 | 54.2 | 54.2 | 47.5 |
| LT Liabilities (E.O.P)       | 0.9  | 0.3  | 0.4  | 0.1  | 5.2  | 5.2  | 2.2  |
| Mean Value of Equity         | 9.9  | 16.3 | 10.1 | 10.7 | 10.8 | 8.0  | 9.6  |

| SDBG Max Volatility          | 27.1 | 25.4 | 18.7 | 26.1 | 24.4 | 31.9 | 32.1 |
| Current Liabilities (E.O.P)  | 22.3 | 26.3 | 27.0 | 33.4 | 60.2 | 60.2 | 57.2 |
| LT Liabilities (E.O.P)       | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
| Mean Value of Equity         | 4.9  | 6.6  | 5.8  | 10.2 | 8.8  | 6.8  | 9.0  |

| LOJ Max Volatility           | 29.1 | 24.2 | 21.7 | 16.2 | 16.2 | 28.0 | 26.8 |
| Current Liabilities (E.O.P)  | 2.1  | 39.7 | 54.0 | 60.6 | 60.8 | 60.8 | 60.8 |
| LT Liabilities (E.O.P)       | 12.0 | 15.1 | 6.1  | 6.6  | 37.4 | 37.4 | 37.4 |
| Mean Value of Equity         | 19.4 | 36.3 | 28.9 | 27.8 | 28.6 | 19.9 | 24.2 |

Notes: Unless otherwise stated all units are in Jamaica Dollar billions. Data for 2010 covers the period January 2010 to June 2010.

III. Methodology

The Merton model is a structural model of bank insolvency, capturing the likelihood that a firm’s assets in the future are likely to fall below its liabilities rendering the institution bankrupt. The value of the firm’s liabilities are also known as the default barrier (DB). The distance to default is a function of the growth in firm’s assets, the volatility of the firm’s assets, as well as the difference between the market value of the firm and the default barrier (see equation 1). The numerator measures the distance between the expected one-year ahead market value of the firm’s assets and the distress barrier while
the denominator is used to scale the numerator with respect to units of standard deviations.

\[ d_2^* = \frac{\ln \left( \frac{V_0}{DB} \right) + \left( \mu - \frac{\sigma^2_A}{2} \right) t}{\sigma \sqrt{T}} \]  

(1)

The distance to default therefore measures the number of standard deviations from the mean before a firm’s assets falls below the default barrier (see Figure 1). Assuming that the natural log of future asset values is distributed normally then the firm’s value can be represented as shown in equation 2 below:-

\[ \ln V_A(t) \sim N \left[ \ln V_A + \left( \mu - \frac{\sigma^2_A}{2} \right) t, \sigma^2_A t \right] \]  

(2)

Equation 1 can be converted into a probability of default, \( N(-d_2^*) \), using the cumulative normal distribution shown in equation 3 (McDonald 2002). That is, the probability that \( V_A(T) < DB \) is as follows:

\[ \text{Probability of Insolvency} = N \left( -\frac{\ln \left( \frac{V_0}{DB} \right) + \left( \mu - \frac{\sigma^2_A}{2} \right) t}{\sigma \sqrt{T}} \right) \]  

(3)

Equity can be viewed as a call option on the value of the firm’s assets (Merton, 1974). Since equity holders have a junior contingent claim on the residual value of assets, the value of equity can be viewed as a call option where holders of equity receive the maximum of either assets minus the default barrier, or nothing in the case of default. That is, equity holders are the residual claimants to the firm’s assets and are only subject to limited liability when the firm is bankrupt. Given that the firm’s equity behaves like a call on the firms’ assets, the Black-Scholes-Merton (BSM) model can be used to compute the (unobservable) value of the firm (\( V_A \)) and the (unobservable) volatility of the firm’s
Figure 1: Merton’s Structural Model of Bank Insolvency

assets ($\sigma_v$) as well as the growth rate of assets $\mu$. These parameters can be deduced by solving the BSM model given the volatility in equity ($\sigma_e$), the market value of the firm’s equity ($V_E$) and the default barrier of the firm (DB) over a given time horizon ($T$). The BSM equation for valuing equity as a European call option on the value of the firm’s assets is shown in equation 4.

$$V_E = V_A N(d_1) - DB e^{-rT} N(d_2) \quad (4)$$

where $N(d_1)$ and $N(d_2)$ are the standard cumulative normal of $d_1$ and $d_2$, respectively, and

$$d_1 = \frac{\ln\left(\frac{V_A}{DB}\right) + \left(r + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}} \quad (5)$$

and

$$d_2 = d_1 - \sigma_A\sqrt{T} \quad (6)$$
The value of assets, $V_A$, asset volatility, $\sigma_A$, are estimated by solving simultaneously the call option formulation in equation (4) and the optimal hedge equation shown in equation 5.\(^{14}\)

$$V_E = \frac{\sigma_A N(d_1)V_A}{\sigma_E}$$

(7)\(^{15,16}\)

Finally, the expected return on assets, $\mu$, is computed using equation 6.

$$\mu(t) = \max \left[ \frac{V_A(t)-V_{A,t-1}}{V_{A,t-1}}, r \right]$$

(8)

were $r$ is continuously compounded risk-free rate.

This framework is general enough to be applied in the assessment of insolvency risks for corporates, banks as well as sovereigns.\(^{17}\) Indeed, what changes is merely what constitutes the default barrier (DB) and the market value and volatility of the entity’s ‘equity.’ Specifically, for financial institutions the DB is determined as a function of the short-term and long-term liabilities of the entity, $r$ is the one-year Treasury Bill rate, and the market value of the firms’ equity, $V_E$, is set equal to the market value of the firm’s equity based on the closing price at the end of the evaluation period and the number of stocks outstanding. $T$ is set equal to one year so that the probability emerging out the assessment is the one-year ahead probability of default on an \textit{ex ante} basis.

Unlike Gapen, \textit{et al.} 2004 which treat each industry sector as if it were one large firm. As pointed out in Gapen, \textit{et al.} (2004), a negative feature of aggregating across industry or sector is that it may be possible for the strength of one entity to offset the weakness in another in ways that do not reflect the underlying and systematic risk exposure of the

\(^{14}\) Recall, that the market value of assets, $V_A$, asset volatility, $\sigma_A$, and the expected return on assets, $\mu$, have to be estimated since these values are not directly observable.

\(^{15}\) See Appendix A for the GAMS programme used to solve for the market value of the firm’s assets and the volatility of those assets.

\(^{16}\) Equation 7 shows the relationship between volatility of the firm’s assets and the volatility of the firm’s equity where $N(d1)$ is the change in the price of equity with respect to a change in the underlying assets of the firm.

\(^{17}\) The most popular commercial product is the KMV model.
industry or sector in times of stress. Hence, this paper in contrast computes the distance to default for each firm within an industry (DTIs and NBFIs) and then uses the market values derived from option pricing theory to weight each observation to get a single index of systemic risk for each industry. Also, the distance-to-distress for each institution is computed and the inter-quartile range is used to derive an alternative systemic risk index for both DTIs and NBFIs over the period under review.

Following closely the methodology used in Gray, Merton and Bodie (2003) and Gapen et al. (2004), this framework can be altered to assess the likelihood of a sovereign default. As discussed earlier, to the extent that the corporate sector holds government debt directly, any severe public sector distress is transmitted to the asset side of the balance sheets of the financial and corporate sectors (Gapen, et al. 2004). The main elements of the asset side of the public sector balance sheet include international reserves, the net present value of primary surpluses and the public sector’s monopoly on the issuance of money.\(^{18}\) The liability side of a sovereign’s balance sheet consists of domestic currency liabilities (domestic currency debt and base money) and foreign currency debt. Many of the assets on the balance sheet of a sovereign, with the possible exception of international reserves, are not traded, and if they are can only be observed at infrequent intervals.

However, the CCA approach described above can be used to impute the value and volatility of a sovereign’s assets. For a sovereign nation, both the short-term foreign-denominated debt and long-term foreign denominated debt make up the DB. Changes in the DB for the public sector come from two sources: (i) changes in the liability structure of external debt from changes in the maturity structure of foreign currency debt, and; (ii) currency movements for the foreign currency debt. On the other hand, the local currency liabilities made up of base money and domestic currency debt behave like a call-option on the Government’s assets. If a sovereign’s assets falls below the level required to cover its foreign currency debt payments, then default occurs. Hence, the value of domestic currency liabilities (base money and domestic debt) can be viewed as a call option on

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\(^{18}\) These assets are net of any guarantees the public sector may implicitly or explicitly provide to the private sector.
sovereign assets with a strike price equal to the level of the distress barrier.\textsuperscript{19} Thus the framework outlined above can be used to determine the implied asset value of a sovereign nation and the volatility of those assets, which in turn can be used to estimate the probability of default.\textsuperscript{20}

Finally, the framework can also be used to determine the implicit or explicit guarantee that the government provides to the financial sector. The government can be modeled as holding a put option whose value is directly related to the implicit guarantees it provides to safeguard the integrity of the financial system. That is, the government is said to be the holder of risk debt, since they are ‘obligated’ to absorb losses in the event of default since debt holders receive assets of the defaulted firm (or equivalently, the assets of the firm get ‘put’ to the debt holders).\textsuperscript{21} The value of the implicit put option is shown in equation 9.

\[
P = DBe^{-rT}N(-d_2) - AN(-d_1)
\] (9)

The value of this implicit guarantee is solved in a two step process. Firstly, the observed market value of equity of the financial firms and their respective distress barrier’s are used with the call option formula to derive the value of the financial firm’s assets. Second, the marked-to-market value of the financial sector’s assets and the DB are then used with the put option formula in equation (9) to derive the implied market value of risky debt. The sum of the implicit put options for DTIs results in a systemic measure of the expected losses that the government guarantees over a one-year time horizon. The value of the put option requires an assumption over recovery rates, post-financial sector

\textsuperscript{19} The holders of domestic currency liabilities receive the maximum of either sovereign assets minus the distress barrier, or nothing in the case of default.

\textsuperscript{20} This approach assumes that foreign currency debt is senior to local currency debt. That is, governments in distress situations are more likely to ‘dilute’ the holders of local currency through inflation or some restructuring exercise, before defaulting on foreign currency debt.

\textsuperscript{21} Holders of risky debt receive either the default-free debt value or, in the event of default, the senior claim of assets. Since the value of default-free debt is equal to the distress barrier and the implicit put option on the assets of the firm yields \(\max[DB - A, 0]\), the market value of risky debt can be modeled as, \(D = \min[A, DB] = DB - \max[DB - A, 0]\).
bail-out which is normally less than 100.0 per cent. For this analysis, a recovery rate of 80.0 per cent was assumed for the banking sector.\textsuperscript{22}

IV. Results

Government of Jamaica Distance-to-Default and Probability of Default Experience in the Context of the Jamaica Debt Exchange

Prior to 2008, Government of Jamaica’s (GOJ) assets were on average three-and-a-half standard deviations away from its distress barrier, peaking in August of 2007 at 4.3. This distance to default correlated with low levels of probability of default over a one-year ahead time horizon. However, in the aftermath of the September 2008 global financial sector meltdown both the domestic non-bank financial sector and the Jamaican Government faced external funding shortfalls. This, in addition to the fall-out in the main sources of foreign currency earnings also resulted in increased pressure on the domestic currency. In response, the Bank of Jamaica intervened in the foreign exchange market and provided liquidity to the non-bank sector and the inter-bank market. The Bank also tightened its monetary policy stance sharply through higher cash reserve requirements and a 680 basis point increase in the policy rate to 21 1/2 per cent on 1\textsuperscript{st} December 2008.

By the second half of 2009, fear over the sustainability of fiscal policy in a context of the closure of international capital markets combined with a weakening domestic economy caused further deterioration in financial market conditions in Jamaica. The Jamaica Dollar depreciated to J$89.05 per U.S. dollar in June 2009 from J$71.56 per U.S. dollar in September 2008 before ending the year at J$89.60.\textsuperscript{23}

\textsuperscript{22} Based on this structure, declines in the value of the financial sector equity and increases in loan delinquency rates causes the market value of assets to decline, increasing the probability of default. As the probability of default rises, the value of the government guarantee adjusts for recovery rates increases.

\textsuperscript{23} In the aftermath of the September 2008 global financial sector meltdown both the domestic non-bank financial sector and the Jamaican Government faced external funding shortfalls. This in addition to the fall-out in the main sources of foreign currency earnings resulted in increased pressure on the domestic currency. In response, the Bank of Jamaica intervened in the foreign exchange market and provided liquidity to the Securities Dealers and the inter-bank market.
At end-November 2009, approximately 40.0 per cent of the total domestic debt was maturing in less than 24 months. Of that 40.0 per cent, which would mature in less than 24 months, 55.0 per cent was variable rate debt. This presented significantly high levels of roll-over risk over the short-term and left the Government vulnerable to sudden adverse shifts in market sentiments. As a direct result, interest payments as a ratio of GDP had tripled relative to the preceding five years, with interest payments accounting for 23.5 per cent of GDP at end-2009, reflecting an average of 60.0 per cent of Government revenues annually.24,25

The CCA provides some indication of these vulnerabilities prior to the financial market volatility that would ensue with the distance-to-default falling precipitously to two standard deviations by end-2008 (see Figures 2 and 3). The value of equity in the public sector began to decline in early 2009, further reducing the distance to distress for the GOJ. Additionally, the increased volatility in the exchange rate and the reduction in the NIR served to reduce the value of the GOJ's assets. All this culminated in the distance to default decreasing to one and the corresponding one-year-ahead probability of default rising to 14.0 per cent by end-March 2009.

By the end-2009 the GOJ was in advanced stages of negotiating a Stand-by Arrangement with the International Monetary Fund (IMF) and other Multi-lateral Financial Institutions (MFIs) for balance of payments support of USD$2.4 billion. The GOJ also undertook a significant debt re-profiling exercise of its domestic debt of $700.0 billion or 65.0 per cent of GDP in January 2010.

24 These high and rising levels of debt service costs inhibited investment in infrastructure and other essential services, generated excessive high real and nominal interest rates and catalyzed recurring fiscal slippages.
25 An increase in exchange rate volatility increases the volatility of assets for the government which reduces the distance to default and increases the default probability for the Government. Similarly, an increase in domestic interest rates reduces the sovereign’s assets which decreases the distance to default and increases the probability of default. A reduction in reserves reduces the value of Assets and has the same effect.
The transaction targeted 100.0 participation of domestic bond holders, with the aim of doubling the average age of the domestic debt profile while lowering the interest costs of GOJ by an average of 850 basis points. The GOJ also took many steps to address the market uncertainty, focusing on core policies that would entrench fiscal discipline and
restore market confidence. Successful implementation of the Jamaica Debt Exchange (JDX), the signing of an IMF agreement by February 2010 and improving fundamentals served to increase the distance to default to four standard deviations by end-March 2010. The mapping of the distance to default into probability of default indicated that at end-March 2010 the one-year ahead probability of default had declined to 0.001 per cent.

**Non-Bank Financial Institutions**

Macro-economic weaknesses in the Jamaican economy and increasing uncertainty in the vulnerability of domestic financial institutions were transferred to the balance sheets of the financial sector, reflecting itself in lower equity prices, increased asset volatility and decreasing distance to distress. The non-bank financial sector was particularly vulnerable as a direct consequence of (i) their large holdings of GOJ securities, (ii) the short-term nature of their funding base, and; (iii) their leveraged investment positions in GOJ global securities. Equity valuations began to decline in September 2008 and continued to decline on a near-continuous basis before bottoming out in March 2009 (see **Figure 4**).

**Figure 4. Value of Equity for Non-Bank Financial Institutions (NBFIs)**

![Graph shows the value of equity for non-bank financial institutions from June 2008 to June 2010.](image)

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26 As the market value of equity declines then the firm has less of an equity cushion to fund its liabilities.
At end-March 2009 the value of equity for NBFIs since the collapse of Lehman Brothers at end-September 2008 had declined by 44.8 per cent to J$39.4 billion. In addition, the default barrier for NBFIs increased by 41.3 per cent to J$353.4 billion at end-2008 relative to end-2007. As equity valuations declined significantly and the default barrier increased, the distance-to-default for many firms in the NBFI sector decreased precipitously (see Figure 5).

Figure 5. Distance to Default for Non-Bank Financial Institutions (NBFIs)

Figure 6. Probability of Insolvency for Non-Bank Financial Institutions (NBFIs)
At the height of the macro-financial uncertainties, the 25\textsuperscript{th} percentile and 75\textsuperscript{th} percentile of the distance to default for NBFIs ranged from 0.37 standard deviations to 4.63 standard deviations, respectively. This distance-to-default mapped to a one-year probabilities of default of 24.1 per cent for the NBFI sector at end-2009 when financial uncertainties were peaking (see Figure 6).

Deposit Taking Institutions (DTIs)

In spite of the increasing volatility in financial markets and deteriorating macro-economic conditions which prevailed in the Jamaican economy after the collapse of Lehman Brothers the equity market was selective in terms of sectors for which it had a negative outlook. In particular, the equity price declines were more significant for non-bank financial institutions relative to their banking counterparts. For the period end-September 2008 to end-March 2009 the value of equity for DTIs declined by 37.0 per cent. While over the same period, the volatility of equity increased by five percentage points to 16.0 per cent. In spite of this, the marked-to-market value of the assets of DTIs continued to grow peaking at $543.4 billion at end-March 2009, representing an annual growth of 16.7 per cent (see Figure 7).\textsuperscript{27}

This positive outlook on the valuation on banks may have reflected lower risk aversion of investors to DTIs as a result of the access to deposit insurance that depositors had via the Jamaica Deposit Insurance Corporation (JDIC) as well as possible the implicit guarantee from the GOJ for the solvency of systemically important financial institutions. Figure 8 plots the evolution of the one-year ahead estimate of the value of the financial sector guaranteed over the past five years. As shown in the figure, the value of the guarantee was approximately 50.0 per cent of nominal GDP at end-2009. The CCA estimated the value of the financial sector guarantee at between 69.1 per cent and 45.3 per cent of GDP over the period.\textsuperscript{28,29}

\textsuperscript{27} In contrast, the marked to mark value of assets for the non-bank sector declined by 4.75 per cent between December 2008 and December 2009.

\textsuperscript{28} This compares with the historical estimate of 40.0 of GDP based on the financial sector turmoil of the 1990s.
The increase in the market value of the financial sector assets relative to the distress barrier decreased the value of the financial sector guarantee.

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29 The increase in the market value of the financial sector assets relative to the distress barrier decreased the value of the financial sector guarantee.
Outside of these factors, the banking sector weathered the financial volatility associated with the global financial turmoil by adjusting its business model. At the height of financial uncertainties banks sold much of their holdings of foreign currency GOJ securities and tilted their portfolio towards domestic currency GOJ securities and loans to the private sector which served to hedge their balance sheets from deteriorations in bond prices for GOJ globals after the collapse of Lehman Brothers.

Consequently, although the inter-quartile range for distance to default for DTIs showed a steady decline since September 2008, by end-December 2009 the inter-quartile range for the distance for default moved from 3.0 standard deviations to 4.95 standard deviations, respectively. This reflected a benign outlook of the risk of insolvency of the banking sector of 0.01 per cent at end-2009 (see Figure 9). Therefore equity markets discriminated against non-banks in favour of deposit-taking institutions up until the end of 2009. However, by March 2010 the probability of default for the banking sector showed an uptick for the first time since the global financial turmoil registering a one-year ahead probability of default of 5.0 per cent (see Figure 10). This increase reflected the deterioration in the growth rate of the marked-to-market value of its assets, on the one hand, and deteriorating prospects for future profitability, on the other. The sharp drop in external demand emanating from the global financial distress had begun to impair the ability of borrowers to service their loans which was manifested in rising non-performing loans ratios for the banking sector. Further, the JDX transaction, which served to reduce the interest cost of the GOJ, meant that banks would have to absorb the impact of lower interest spreads between loans and deposits.  

30 Against this background, the Financial System Stability Fund (FSSF) of approximately US$950.0 million was established in February 2010, funded by resources from the IMF, the World Bank and the IDB as a part of the agreement emanating from the successful execution of the JDX. These funds would be accessible to participating financial institutions in the event of, for example, a margin call on funds borrowed from overseas banks arising directly from the debt swap, a liquidity run on an institution, as well as those arising from liquidity mismatches emanating from the transaction.

31 A reduction in the stock market resulting in a results the marked to market assets of the corporate sector which in turn leads to a reduction in the assets of banks and leads to an increase in the implicit guarantee of the financial sector by the Government. This leads to reduction in the assets of the sovereign which decreases the D2D and increases the Default Probability.
VI. CONCLUSION

The paper explores the use of the contingent claims approach (CCA) to the estimation of bankruptcy risk of both financial institutions and the Government of Jamaica. This framework can be utilized regulators as a forward-looking high frequency gauge of the potential build up in risks to the sovereign as well as the financial system. The framework also estimates the associated value of macro-financial risk transfers across interrelated
balance sheets of the financial and public sector so these risk transfers can be monitored and policy adjusted accordingly. Finally, the framework incorporates the impact of a full or partial financial guarantee from the government to the financial sector as well as means of evaluating how changes in the solvency of the government affect the viability of the financial sector in Jamaica. The CCA framework thus provides an interconnected framework within which policy makers can analyze the impact of potential policy mixes on financial system stability and evaluate which options may be more suitable at countering emerging vulnerabilities.

Applying the CCA framework retroactively to both the sovereign GOJ balance sheet and the balance sheet of the financial sector suggests that this approach would have provided an accurate view of the pending domestic macro-economic and financial turmoil in the wake of the global financial distress. The results show that the global financial crisis had its first impact primarily on the central government via the closure of access to global financial markets as well as significant increases in the financing costs of new debt raising initiatives domestically. The risks then transferred to the asset side of the financial sector, which held GOJ securities via significant declines in the marked-to-market value of its holders of global GOJ bonds as well as margin calls sustained after the collapse of Lehman Brothers in September 2008. The assessment highlights the pivotal role that the successful execution of the JDX as wells as the signing of the IMF agreement played in mitigating the macro-financial risks that had become self-evident by end-2009.

**Bibliography**


